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# EXACT SOLUTION OF THE TRANSIENT FORCED CONVECTION ENERGY EQUATION FOR TIMEWISE VARIATION OF INLET TEMPERATURE

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**Abstract**—An exact solution to the equation of transient forced convection for time varying inlet temperature with a general, space dependent boundary condition of an incompressible laminar forced convection heat transfer with fully developed flow between two parallel plates is given. The finite integral transform technique has been used as the method of analysis. Analytical results for laminar and turbulent flow are presented. The results are confirmed experimentally by the frequency response method.

## NOMENCLATURE

- $D$, effective diffusivity, $(= a + \varepsilon_h)$
- $D_e$ equivalent diameter, $(= 2d)$;
- $T$, temperature;
- $a$, thermal diffusivity $(= k/\rho c_p)$;
- $c_p$, specific heat at constant pressure;
- $d$, half distance between parallel plates;
- $k$, thermal conductivity;
- $t$, time;
- $u$, velocity component in-$x$ direction;
- $\bar{u}$, average velocity;
- $x, y$, cartesian coordinates ($x$-flow direction, $y$-distance from duct centerline);
- $Q$, dimensionless temperature $[= (T - T_0)/(\Delta T)_0]$;
- $\rho$, fluid density;
- $Pr$, Prandtl number $(= c_p\mu/k)$;
- $Re$, Reynolds number $(= 2\bar{u}d/\nu)$;
- $X$, dimensionless distance along the duct $(= x/D_e)$;
- $\alpha$, parameter for laminar flow $(= a\lambda_n^2/\bar{u})$, $\delta(= \beta/\bar{u})$;
- $\bar{\alpha}, \bar{\delta}$, parameters for turbulent flow.

Meaning of any other symbols are given in the text as they occur.

## 1. INTRODUCTION

THE STUDY of unsteady forced convection heat transfer in tubes and ducts has recently become of greater importance in connection with the control of modern high performance heat transfer devices. Literature on thermal transients is limited but increasing. Some of the important contributions are listed in the references [1–22]. In solutions of the problem of transient forced convection in laminar flow, it has usually been assumed that the inlet temperature of the fluid is constant across the flow with a specified timewise variation of wall temperature, wall heat flux or internal heat generation. There is also some work done on the thermal transient problems in heat exchangers: the response of a fluid flowing steadily through an insulated pipe subjected to a step increase in the inlet temperature has been published by Rizika [8, 9] for both compressible and incompressible systems. A numerical method for calculating heat exchanger dynamics was given by Dusinberre [10] who has presented explicit iteration formulas and computation guides. In [18], a specified consideration is given to laminar flow in a parallel plate channel for time varying inlet temperature and participating walls.

The general problem of transient forced convection heat transfer may be stated as follows; the temperature distribution is to be determined in the system at an arbitrary instant of time, given:

(a) The inlet temperature distribution as an arbitrary function of time and space.
(b) Initial temperature distribution for $x > 0$ as an arbitrary function of time and space.
(c) A prescribed boundary condition which may take many forms. Some possible forms are described below
   —A prescribed temperature distribution or heat flux distribution may in some way be enforced on the boundaries of the system, and this distribution may furthermore be constant or variable with time and/or space.
   —A constant heat transfer coefficient to a prescribed ambient temperature.

In the present analysis a general solution is presented for laminar as well as turbulent flow in a parallel plate channel under a prescribed boundary condition with an inlet temperature which varies sinusoidally in time. Experimental results for the lowest eigen-value for turbulent flow are presented.

## 2. FORMULATION OF THE PROBLEM

Consideration is given to a parallel plate channel whose sides are separated by a distance

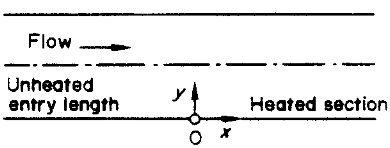


FIG. 1.

$2d$. A steady laminar flow passes through the channel. The fluid entering the channel has a temperature which is spacially uniform across the entrance section but varies sinusoidally with time. Therefore we can write the inlet condition

$$T(0, y, t) = T_0 + (\Delta T)_0 \sin \beta t \qquad (1)$$

where $T_0$ is the cycle mean temperature, $(\Delta T)_0$ is the amplitude and $\beta$ is the inlet frequency.

The parallel plate channel under consideration is shown in Fig. 1. Axial distances from the entrance section are measured by coordinate-$x$, while the transverse distances are measured by-$y$ (duct centerline corresponds to $y = 0$).

Starting point of the analysis is the unsteady-state energy equation for a fully-developed hydrodynamic flow in a parallel sided duct.

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} = a\frac{\partial^2 \theta}{\partial y^2} \qquad (2a)$$

where

$$\theta(x, y, t) = \frac{T(x, y, t) - T_0}{(\Delta T)_0}.$$

The system satisfying equation (2a) is subjected to the following restrictions:

(a) Fully developed laminar velocity profile between the parallel plates.
(b) Frictional dissipation of energy is negligible.
(c) Axial conduction is negligible with respect to bulk transport in the $x$-direction. This is a reasonable assumption when Péclét number exceeds 100 [20].
(d) Fluid property variations are also neglected.
(e) Thermal resistance of the channel wall is negligible.

The inlet and the boundary conditions of the problem can be written

$$\theta(0, y, t) = \sin \beta t \qquad (2b)$$

$$\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = 0, \left[k\frac{\partial \theta}{\partial y} + h\theta\right]_{y=d} = f(x)$$

$$t > 0. \qquad (2c, d)$$

One recovers the temperature boundary condition at the outer boundary by setting $k$ equal to zero and $h$ equal to one and heat flux boundary condition by setting $h$ equal to zero. When $h$ and $k$ are finite, equation (2d) means that the outer boundary loses heat by convection.

## 3. SOLUTION

The foregoing problem can be separated into two as follows

$$\theta(x, y, t) = \theta_1(x, y) + \theta_2(x, y, t) \quad (3)$$

where the new temperature functions satisfy the following problems

$$\frac{\partial \theta_1}{\partial x} = \frac{a}{u}\frac{\partial^2 \theta_1}{\partial y^2} \quad (4a)$$

$$\theta_1(0, y) = 0 \quad (4b)$$

$$\left(\frac{\partial \theta_1}{\partial y}\right)_{y=0} = 0, \left[k\frac{\partial \theta_1}{\partial y} + h\theta_1\right]_{y=d} = f(x) \quad (4c, d)$$

$$\frac{\partial \theta_2}{\partial t} + u\frac{\partial \theta_2}{\partial x} = a\frac{\partial^2 \theta_2}{\partial y^2} \quad (5a)$$

$$\theta_2(0, y, t) = \sin \beta t \quad (5b)$$

$$\left(\frac{\partial \theta_2}{\partial y}\right)_{y=0} = 0, \left[k\frac{\partial \theta_2}{\partial y} + h\theta_2\right]_{y=d} = 0 \quad t > 0. \quad (5c, d)$$

In obtaining $\theta_2$, firstly it will be assumed that $h \neq 0$. We define the following auxiliary problem

$$\frac{\partial \bar{\theta}_2}{\partial t} + u\frac{\partial \bar{\theta}_2}{\partial x} = a\frac{\partial^2 \bar{\theta}_2}{\partial y^2} \quad (6a)$$

$$\bar{\theta}_2(0, y, t) = \cos \beta t \quad (6b)$$

$$\left(\frac{\partial \bar{\theta}_2}{\partial y}\right)_{y=0} = 0, \left[k\frac{\partial \bar{\theta}_2}{\partial y} + h\bar{\theta}_2\right]_{y=d} \quad (6c, d)$$

It is to be noted that the auxiliary problem is similar to the original problem for $\theta_2$ except that the periodic condition has a shift by $\Pi/2$.

If we define a new temperature function $\theta_c(x, y, t)$ such that

$$\theta_c = \bar{\theta}_2 + i\theta_2 \quad (7)$$

then the problems given by equations (5) and (6) can be combined to get the following problem

$$\frac{\partial \theta_c}{\partial t} + u\frac{\partial 0}{\partial x} = a\frac{\partial^2 \theta}{\partial y^2} \quad (8a)$$

$$\theta_c(0, y, t) = e^{i\beta t} \quad (8b)$$

$$\left(\frac{\partial \theta_c}{\partial y}\right)_{y=0} = 0, \left[k\frac{\partial \theta_c}{\partial y} + h\theta_c\right]_{y=d} = 0 \quad t > 0. \quad (8c, d)$$

A periodic solution of the following type can be assumed

$$\theta_c(x, y, t) = e^{i\beta t}\,\xi(x, y). \quad (9)$$

Introducing the definition given by equation (9) into equation (8a), we get

$$\frac{\partial^2 \xi}{\partial y^2} - \frac{u}{a}\frac{\partial \xi}{\partial x} - i\frac{\beta}{a}\xi = 0. \quad (10a)$$

Boundary and inlet conditions for this problem becomes

$$\xi(0, y) = 1 \quad (10b)$$

$$\left(\frac{\partial \xi}{\partial y}\right)_{y=0} = 0, \left[k\frac{\partial \xi}{\partial y} + h\xi\right]_{y=d} = 0. \quad (10c, d)$$

Now to solve the problems given by equations (4) and (10), we define the following eigen-value problem

$$\frac{d^2 Y_n}{dy^2} + \lambda_n^2 Y_n = 0 \quad (11a)$$

$$\left(\frac{dY_n}{dy}\right)_{y=0} = 0, \left[k\frac{dY}{dy} + hY_n\right]_{y=d} = 0. \quad (11b, c)$$

From boundary condition (11b) we conclude that eigen-functions are $\cos \lambda_n y$. These eigen-functions form an orthogonal set, in the sense that an arbitrary function $\Psi(x, y)$ can be expanded in terms of the eigen-functions.

*Table 1. Eigen-values*

| Boundary condition at $y = d$ | Eigen-values |
|---|---|
| 1st kind ($k = 0$, $h = 1$) | $\lambda_n = \frac{2n-1}{d}\frac{\Pi}{2}$, $n = 1, 2 \ldots$ |
| 2nd kind ($h = 0$) | $\lambda_n = \frac{n-1}{d}\Pi$, $n = 1, 2 \ldots$ |
| 3rd kind ($h$ and $k$ are finite) | Positive roots of $\lambda_n \tan \lambda_n d = h/k$ |

$$\Psi(x, y) = \sum_{n=1}^{\infty} B_n(x) \cos \lambda_n y. \quad (12)$$

Eigen-values are given in Table 1.

The expansion coefficients $B_n(x)$ can be determined by utilizing the orthogonality property of the eigen-functions, the result is

$$B_n(x) = \frac{\int_0^d \Psi(x, y) \cos \lambda_n y \, dy.}{\frac{d}{2} + \frac{1}{4\lambda_n} \sin 2d\lambda_n}. \quad (13)$$

After substituting $B_n(x)$ into equation (12) if we define a new function $\overline{\Psi}(x)$

$$\overline{\Psi}(x) = \int_0^d K_n(y)\, \Psi(x, y)\, dy \quad (14)$$

then the function $\Psi(x, y)$ can be written as

$$\Psi(x, y) = \sum_{n=1}^{\infty} K_n(y)\, \overline{\Psi}_n(x) \quad (15)$$

where we have defined

$$K_n(y) = \frac{\cos \lambda_n y}{\sqrt{\left(\frac{d}{2} + \frac{1}{4\lambda_n} \sin 2d\lambda_n\right)}}. \quad (16)$$

Here equation (14) is the finite integral transform of $\Psi(x, y)$ and equation (15) is the corresponding inversion formula.

To simplify the method of analysis the case of constant velocity will be treated here, and for this purpose we substitute $\bar{u}$ (= mean velocity) for the velocity profile. After this change we take the transform of the differential equations (4a) and (10a) according to the definitions given by equation (14) to yield

$$\frac{d\bar{\theta}_{1n}}{dx} = \frac{a}{\bar{u}} \int_0^d \frac{\partial^2 \theta_1}{\partial y^2} K_n(y)\, dy \quad (17)$$

$$\int_0^d \frac{\partial^2 \xi}{\partial y^2} K_n(y)\, dy - \frac{\bar{u}}{a} \frac{d\bar{\xi}_n}{dx} - i\frac{\beta}{a} \bar{\xi}_n = 0. \quad (18)$$

The integrals in equations (17) and (18) can be performed by integrating them by parts, by utilizing the eigen-value problem given by equations (11) and the boundary conditions to yield

$$\frac{d\bar{\theta}_{1n}(x)}{dx} + \lambda_n^2 \frac{a}{\bar{u}} \bar{\theta}_{1n}(x) = A(x) \quad (19)$$

$$\frac{d\bar{\xi}_n(x)}{dx} + \left(\lambda_n^2 \frac{a}{\bar{u}} + i\frac{\beta}{\bar{u}}\right) \bar{\xi}_n(x) \quad (20)$$

where the function $A(x)$ is given in Table 2.

*Table 2. Function* A(x)

| Boundary condition at $y = d$ | $A(x)$ |
|---|---|
| 1st kind ($k = 0$, $h = 1$) | $-\frac{a}{\bar{u}} f(x) \left(\frac{dK_n}{dy}\right)_d$ |
| 2nd and 3rd kind | $\frac{a}{\bar{u}} f(x) \frac{K_n(d)}{k}$ |

Solution of these two ordinary differential equation become

$$\bar{\theta}_{1n}(x) = e^{-\alpha_n x} \int_0^x e^{\alpha_n z} A(z)\, dz \quad (21)$$

$$\bar{\xi}_n(x) = \bar{\xi}_n(0) \exp\left[-(\alpha_n + i\delta)x\right] \quad (22)$$

where we have defined

$$\alpha_n = \frac{a\lambda_n^2}{\bar{u}}, \delta = \frac{\beta}{\bar{u}} \quad (23a, b)$$

$$\bar{\xi}_n(0) = \frac{\sin \lambda_n d}{\lambda_n \sqrt{\left(\frac{d}{2} + \frac{1}{4\lambda_n} \sin 2d\lambda_n\right)}}. \quad (24)$$

Inverting equations (21) and (22) according to equation (15) we get

$$\theta_1(x, y) = \sum_{n=1}^{\infty} K_n(y)\, e^{-\alpha_n x} \int_0^x e^{\alpha_n z} A(z)\, dz \quad (25)$$

$$\xi(x, y) = \sum_{n=1}^{\infty} K_n(y)\, \bar{\xi}_n(0) \exp\left[-(\alpha_n + i\delta)x\right] \quad (26)$$

and the solution for $\theta_c(x, y, t)$ can be written

$$\theta_c(x, y, t) = \exp\left[i(\beta t - \delta x)\right] \sum_{n=1}^{\infty} e^{-\alpha_n x} K_n(y) \bar{\xi}_n(0). \quad (27)$$

Solution for $\theta_2(x, y, t)$ can be found taking the imaginary part of equation (27)

$$\theta_2(x, y, t) = \sin(\beta t - \delta x) \sum_{n=1}^{\infty} e^{-\alpha_n x} K_n(y) \bar{\xi}_n(0). \tag{28}$$

In obtaining $\theta_2(x, y, t)$, so far it is assumed that $h \neq 0$. When the boundary condition on the wall is of the second kind (e.g. $h = 0$), $\theta_2$ is not a a function of $y$-coordinate, then it can readily be shown that $\theta_2$ becomes

$$\theta_2 = \sin(\beta t - \delta x).$$

Therefore complete solution of the problem can be written

$$\theta(x, y, t) = \frac{T(x, y, t) - T_0}{(\Delta T)_0} = \theta_2(x, y, t) + \sum_{n=1}^{\infty} e^{-\alpha n x} K_n(y) \int_0^x e^{\alpha_n z} A(z)\, dz \tag{29}$$

where

$$\theta_2(x, y, t) = \begin{cases} \sin(\beta t - \delta x) \sum_{n=1}^{\infty} e^{-\alpha_n x} K(y)\, \xi(0), & \text{when } h \neq 0 \\ \sin(\beta t - \delta x), & \text{when } h = 0. \end{cases}$$

$\theta(x, y, t)$ gives the dimensionless temperature distribution between two parallel plates when the inlet temperature has been changed periodically. The solution represents the exact solution of energy equation for slug flow assumption.

When the boundary condition on the wall for $\theta(x, y, t)$ is homogeneous, that is, when the function $f(x)$ is zero, then $\theta_1(x, y)$ is identically zero and in that case we have

$$\theta(x, y, t) = \theta_2(x, y, t). \tag{30}$$

$\theta_2(x, y, t)$ shows that each mode of temperature decays exponentially along the duct, and this decay is inversely proportional to the velocity $\bar{u}$. Therefore for a given flow regime as Reynolds number increases decay decreases. It is also seen that phase lag along the channel is linear and slope of this is $\delta$. Also, as the inlet frequency is increased phase lag increases and as the velocity, $\bar{u}$ is increased $\delta$ decreases. These have also been verified by experiment which is still under further investigations.

The analysis up to now has been for laminar flow, for turbulent flow again the nature of the problem is the same but with different eigenvalues, eigen-functions and expansion coefficients. Therefore we propose a solution of the following type for homogeneous boundary conditions

$$\bar{\theta}_c(x, y, t) = \exp[i(\beta t - \bar{\delta} x)] \sum_{n=1}^{\infty} C_n e^{-\bar{\alpha} x} R_n(y) \tag{31}$$

where $R_n(y)$ and $C_n$ are the new eigen functions and expansion coefficient respectively and $\theta_c$ is the new complex temperature distribution for turbulent flow. $\bar{\delta}$ and $\bar{\alpha}_n$ are new variables for turbulent flow.

The transient energy equation for fully developed turbulent flow for variable effective diffusivity, variable-velocity across the duct, in the absence of frictional dissipation and axial diffusion is

$$\frac{\partial \bar{\theta}_c}{\partial t} + u \frac{\partial \bar{\theta}_c}{\partial x} = \frac{\partial}{\partial y}\left[D(y) \frac{\partial \bar{\theta}_c}{\partial y}\right]. \tag{32}$$

Substitution of equation (31) into equation (32) reveals that $R_n(y)$ should be a complex function, that is

$$R_n(y) = P_n(y) + iQ_n(y). \tag{33}$$

Introducing this definition into equation (32) we get the following two coupled differential equations

$$\frac{d}{dy}\left[D(y) \frac{dP_n}{dy}\right] = -\bar{\alpha}_n u P_n - (\beta - \delta u) Q_n \tag{34a}$$

$$\frac{d}{dy}\left[D(y) \frac{dQ_n}{dy}\right] = -\bar{\alpha}_n u Q_n - (\beta - \delta u) P_n. \tag{34b}$$

For a given flow regime every value of $\beta$

gives rise to a set of eigen-values for $\bar{\alpha}$ and $\bar{\delta}$ and a corresponding set of eigen-functions for $P_n$ and $Q_n$ are also obtained.

As a special case when the velocity $u$ and effective diffusivity $D$ are constants, the eigen functions and eigen-values are found to be

$$P_n = \cos \eta_n y \cos h\mu_n y \tag{35a}$$

$$Q_n = -\sin \eta_n y \sin h\mu_n y \tag{35b}$$

where the eigen value is given by

$$\lambda_n = \eta_n + i\mu_n = \sqrt{[\bar{\alpha}_n \bar{u} - i(\beta - \bar{\delta}\bar{u})]}. \tag{36}$$

In this case $\bar{\alpha}$ and $\bar{\delta}$ is found to be

$$\bar{\alpha}_n = \frac{D(\eta_n^2 - \mu_n^2)}{\bar{u}} \tag{37a}$$

$$\bar{\delta} = \frac{\beta + 2D\mu_n\eta_n}{\bar{u}}. \tag{37b}$$

Thus an exact solution to the transient energy equation for time varying fluid inlet temperature under the space dependent general boundary condition of an incompressible laminar and turbulent heat transfer between two parallel plates is obtained.

## 4. COMPARISON WITH EXPERIMENT

In many applications heat transfer in regions away from the inlet is of interest; for such situations only the first term in the series need to be considered and from the equation (31), the asymptotic solution of the decay of an inlet condition becomes

$$\theta_c(x, y, t) = \exp[i(\beta t - \bar{\delta}x)]\, C_1\, e^{-\bar{\alpha}_1 x} R_1(y). \tag{38}$$

Imaginary part of this solution gives the temperature distribution for turbulent flow under homogeneous boundary conditions.

$$\theta_2(x, y, t) = C_1\, e^{-\alpha_1 x}\, [P_1(y) \sin(\beta t - \bar{\delta}x) + \theta_1(y) \cos(\beta t - \bar{\delta}x)]. \tag{39}$$

This solution can be put into the following form

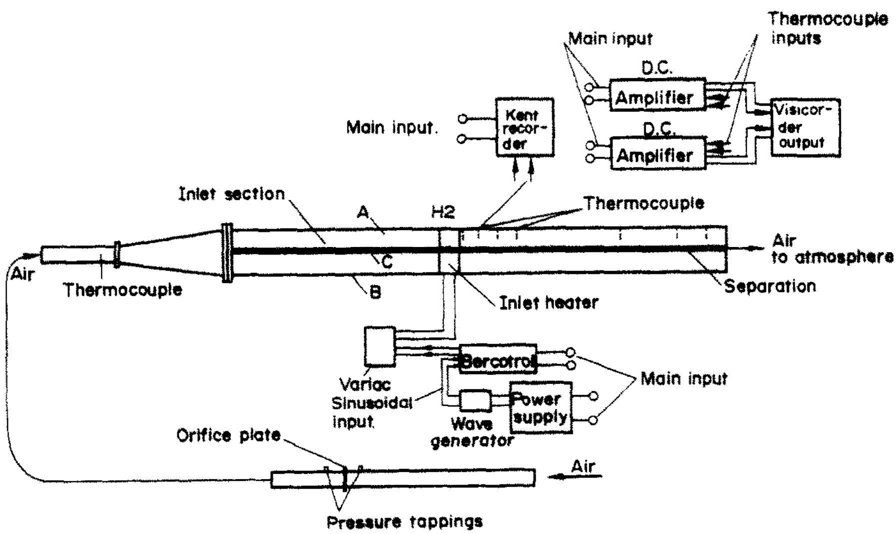


FIG. 2.

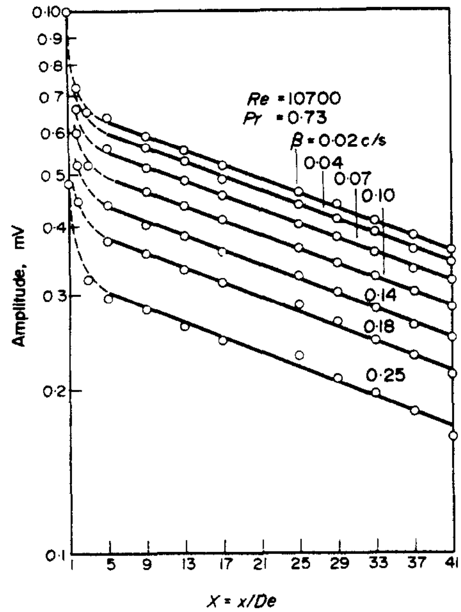


FIG. 3.

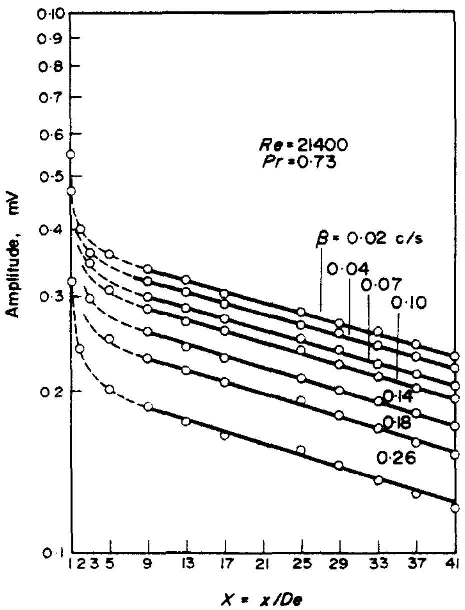


FIG. 4.

$$\theta_2(x, y, t) = C_1 \sqrt{(P_1^2 + Q_1^2)}\, e^{-\bar{\alpha}_1 x} \sin(\beta t - \bar{\delta} x + \varepsilon) \quad (40)$$

where

$$\cos\varepsilon = \frac{P_1}{\sqrt{(P_1^2 + Q_1^2)}}, \sin\varepsilon = \frac{Q_1}{\sqrt{(P_1^2 + Q_1^2)}}.$$

Thus $\varepsilon$ is a function of $y$ only, and is independent of $x$ and $t$.

For a given value of $y$, $P_1$ and $Q_1$ are constants and temperatures at any $y$, (say $y = 0$) are given by

$$\theta(x, 0, t) = A \exp(-\bar{\alpha}_1 x) \sin(\beta t - \bar{\delta} x + \varepsilon) \quad (41)$$

where we have defined

$$A = C_1 \sqrt{[P_1^2(0) + Q_1^2(0)]}. \quad (42)$$

The form of equations (31) and (41) suggest that the results are best confirmed experimentally by the frequency response method. The apparatus and technique used will be described more fully in another paper. In its final form the apparatus were consisted of a duct 1 in. deep, 1 ft wide and 10 ft long. The first 3 ft of the duct formed an entry section in which the velocity profile was established. A nichrome wire heater was placed across the duct at the junction of the entry section and working section. In the working section copper–constanton thermocouples projected to the midpoint of the duct at intervals along its length. The air flow rate was measured by means of an orifice gauge in a length of pipe connected to the inlet section by an adopter section.

The schematic diagram of the apparatus is shown in Fig. 2.

Zero-type boundary conditions cannot be applied at the fluid–wall interface, but must be taken at the outer surface of the wall.

Calculations show that over the frequency range used, the effect of the thickness of the walls on the parameter measured was less than 4 per cent; experimental measurements made near the fluid–wall interface suggest that this figure is an over estimate.

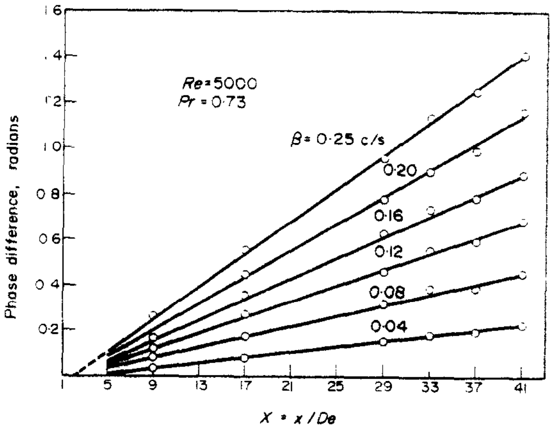


FIG. 5.

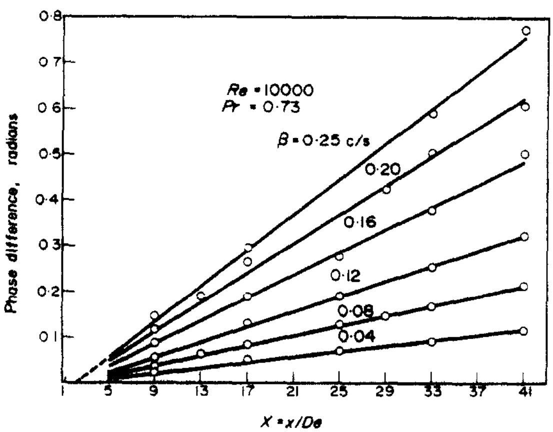


FIG. 6.

The boundary conditions used were zero temperature at one side of the duct and zero heat flux at the other.

A Bercotrol power regulator was used to implify the sinusoidal feed back from the wave generator.

The response to the sinusoidal variation of heat input have been recorded on a strip-chart. From these recordings the amplitudes at various points along the channel have been presented in graphical forms for various inlet frequencies, Figs. 3 and 4.

Phase lags in the response to the sinusoidal variation of heat input have also been determined along the duct for various values of Reynolds numbers and inlet frequencies, Figs. 5 and 6.

As the frequency $\beta$ of the sine wave at the inlet was varied, it was possible from these measurements to obtain the values of $\bar{\alpha}$ and $\bar{\delta}$ for the lowest eigen-function after the higher eigenfunctions excited had damped out. Figs. 7 and 8.

## 5. CONCLUSIONS

Solutions determine the temperature distribution as a function of time and space in the form of infinite series, each term of which includes an exponential term in $x$. This means that each mode of temperature distribution decays exponentially along the duct.

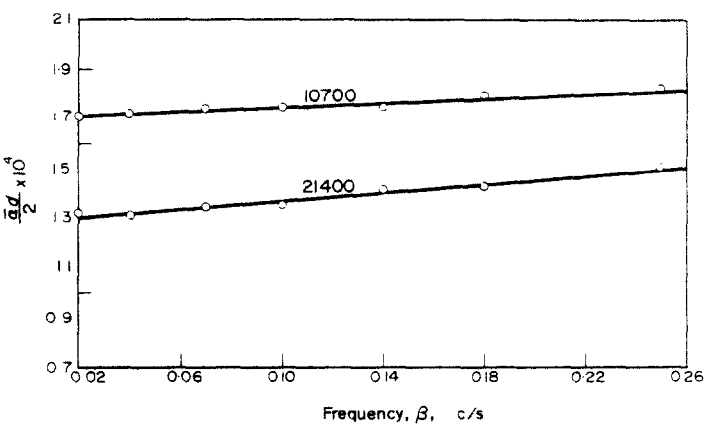


FIG. 7.

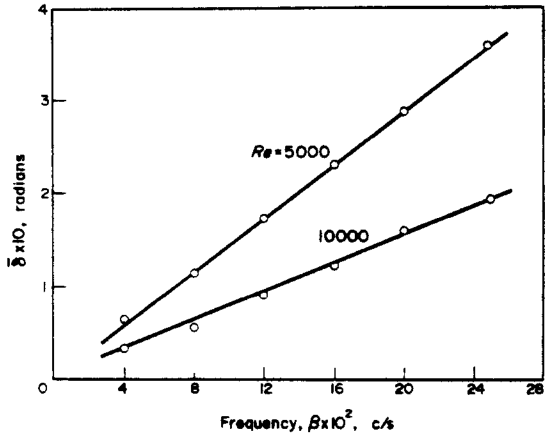


FIG. 8.

From the experimental results, it is seen that the decay of the inlet temperature distribution near the entrance region is not a single exponential. It consists of modes of higher frequency. The basic mode of the inlet temperature varies exponentially along the channel and the value of the temperature at a given point depends on the inlet frequency and Reynolds number. For given fluid as Reynolds number increases decay decreases.


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## REFERENCES


1. M. PERLMUTER and R. SIEGEL, *Trans. ASME* **83**, 432–440 (1961).
2. R. SIEGEL and E. M. SPARROW, *J. Heat Transfer* **80**, 29–36 (1959).
3. R. SIEGEL, *J. Appl. Mech.* **27E**, 241–249 (1960).
4. J. A. CLARK, V. S. ARPACI and K. M. TRENDWELL, *Trans. ASME* **80**, 612–624 (1958).
5. V. S. ARPACI and J. A. CLARK, *Trans. ASME* **80**, 625–634 (1958).
6. V. S. ARPACI and J. A. CLARK, *J. Heat Transfer* **81C**, 253–266 (1959).
7. R. SIEGEL and M. PERLMUTER, *J. Heat Transfer* **85C**, 358 (1963).
8. J. M. RIZIKA, *Trans. ASME* **76**, 411 (1954).
9. J. M. RIZIKA, *Trans. ASME* **78**, 1407 (1956).
10. G. M. DUSINBERRE, *Trans. ASME* **76**, 421 (1954).
11. S. KAKAC, Transient heat transfer by forced convection in ducts, Ph.D. thesis Univ. of Manchester, England (1965).
12. R. G. DEISSLER, *Trans. ASME* **77**, 1221 (1955); or NACATN 3016 (1953).
13. M. N. OZISIK, *Boundary Value Problems of Heat Conduction*. International Textbook Company, Scranton, Pennsylvania (1968).
14. J. L. HUDSON and S. G. BANKOFF, *Int. J. Heat Mass Transfer* **6**, 607 (1963).
15. P. KARDAS, *Int. J. Heat Mass Transfer* **9**, 567 (1966).
16. A. J. GRAM and G. W. KESSLER, *Mech. Engng* **88**(9) (1966).
17. S. KAKAC, *Wärme-Und Stoffübertragung* **1**, 169–176 (1968).
18. E. M. SPARROW and F. N. DE PARIAS. *Int. J. Heat Mass Transfer* **II**, 837–853 (1968).
19. S. KAKAC, Sixth Southeastern Seminar on Thermal Sciences. Raleigh, North Carolina (1970).
20. P. J. SCHNEIDER, *Trans. ASME* **79**, 765–773 (1957).
21. D. B. SPALDING, International Heat Transfer Conference, Boulder, Colorado, Part II, section B pp. 439–446 (1961).
22. D. B. SPALDING, *Int. J. Heat Mass Transfer* **7**, 743–762 (1964).


## SOLUTION TRANSITOIRE EXACTE DE L'EQUATION D'ENERGIE POUR LA CONVECTION FORCEE AVEC VARIATION DE LA TEMPERATURE D'ENTREE


**Résumé**—On donne une solution exacte de l'équation de la convection forcée pour une température d'entrée variable avec une condition aux limites dépendant de l'espace et pour un écoulement incompressible établi entre deux plans parallèles. On utilise la technique de la transformée intégrale. Les résultats analytiques sont présentés pour les écoulements laminaires et turbulents. Ils sont confirmés expérimentalement par la méthode de réponse en fréquence.


## EXAKTE LÖSUNG DER ENERGIEGLEICHUNG FÜR DEN AUSGLEICHSVORGANG BEI ERZWUNGENER KONVEKTION MIT ZEITABHÄNGIGER EINGANGSTEMPERATUR


**Zusammenfassung**—Für den Ausgleichsvorgang bei erzwungener Konvektion mit einer zeitabhängigen Eingangstemperatur wird eine exakte Lösung angegeben, mit einer allgemeinen, raumabhängigen Randbedingung für den Wärmeübergang bei inkompressibler, laminarer erzwungener Konvektion und voll entwickelter Strömung zwischen zwei parallelen Platten. Es wurde die Transformationsmethode für endliche Integrale benutzt. Die analytischen Ergebnisse für laminare und turbulente Strömung werden angegeben. Mit Hilfe der Frequenzgangmethode werden die Ergebnisse experimentell bestätigt.

# ТОЧНОЕ РЕШЕНИЕ УРАВНЕНИЯ ЭНЕРГИИ НЕСТАЦИОНАРНОЙ ВЫНУЖДЕННОЙ КОНВЕКЦИИ ДЛЯ ИЗМЕНЯЮЩЕЙСЯ ВО ВРЕМЕНИ ТЕМПЕРАТУРЫ НА ВХОДЕ

**Аннотация**—В статье приводится точное решение уравнения нестационарной вынужденной конвекции при изменяющейся во времени температуре на входе с граничным условием, зависящим от координат, в полностью развитом течении между двумя параллельными пластинами. При анализе использовался метод конечных интегральных преобразований. Приводятся результаты анализа ламинарного и турбулентного течений. Результаты расчета подтверждаются экспериментами.